MAJOR FLAWS IN THE CURRICULUM DESIGN OF PROJECT MATHS AND THE DIRE CONSEQUENCES FOR IRISH MATHEMATICS EDUCATION AND THE KNOWLEDGE ECONOMY

Background

Mathematics is an underpinning technology for modern society. It is, in fact, the underpinning technology. This is obvious to experts, but less so to the man in the street, who is commonly almost proud of the fact that he 'was never any good at maths'. So the argument will bear indefinite repeating. Put simply, it is hard to think of any technology in which mathematics is not in some way inextricably involved. Weather prediction, climate change, flood prevention; electricity, water supply, sewage treatment; roads, buildings; airline scheduling, supermarket restocking; they all involve mathematics at some fundamental level, but not one which is necessarily visible to the consumer (or indeed the provider). Without mathematics, there would be no computers, no aeroplanes, no space programme. There would be no internet credit card transactions, no Google. There would be no computer games, even if there were computers to run them on. Even if airlines ran hot air balloons, there would be no airlines, because they were would be no way of scheduling arrival and departure of the balloons. Nor indeed would there be train companies or truck companies, not because there would be no trains or trucks (although there wouldn't be), but for the same reason. Almost all science is built around the concept of a mathematical model; a proper mathematical training at all educational levels is a *sine qua non* for a technological or "smart" economy.

What is Project Maths?

From the web site of the National Council for Curriculum and Assessment (NCCA, `www.ncca.ie`), we read that:

> The NCCA conducted a review of post-primary mathematics education. This review came at a time when there was concern about the uptake of Higher level mathematics, particularly in the Leaving Certificate, and about the standards of mathematical achievement in state examinations and in international tests such as the Programme for International Student Assessment (PISA).

> Since mathematics underpins many other disciplines, including science and technology, a decline in mathematical knowledge and skills among school leavers can affect the potential of our society for future economic growth and development. The review also took place against the background of a revised curriculum in primary schools and revised Junior Certificate mathematics syllabuses. Mathematics forms a significant part of a student's education, with all students taking this subject to Leaving Certificate. It is critical, therefore, that issues related to post-primary mathematics education can be widely discussed, and that all voices can have the opportunity to contribute to shaping its future development.

In addition, on `www.projectmaths.ie` it is reported that:

> Project Maths aims to provide for an enhanced student learning experience and greater levels of achievement for all. Much greater emphasis will be placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience.

What is wrong with project Maths?

So far so good. Project Maths appears to be a laudable attempt to update and modernise the Leaving Certificate syllabus: the NCCA clearly acknowledges the importance of mathematics to science and technology. The trouble is that the practice is different: *Project Maths contains serious flaws in its syllabus and methodology.* Project Maths appears to have started as a pilot programme but, in a very Irish way, it has been passed as fit for purpose without any objective evaluation or consultation. Similar schemes have been tried around the world: the results are not encouraging. In this country we do not yet seem to have developed the knack of observing what happens elsewhere, evaluating it, and improving or discarding as required.

The Finnish view

Typical propaganda (on `www.educationmatters.ie`) runs as follows:

> Good practice in mathematics teaching in other countries was also examined. The focus was on the countries that perform better in international studies (PISA,

> TIMSS). These countries are Finland, South Korea, Holland and Japan. NCCA has brought together best practice from around the world and tailored it for the modern Irish school. We have retained what is seen as valuable from the past and blended it with what is viewed as essential for the future.

This sounds reasonable but, in a personal communication, Professor Olli Martio, (Secretary General Finnish Academy of Science and Letters) informs us that:

> *It seems that a new math curriculum for schools has created a major debate in Ireland. The same phenomenon has happened in many countries. Usually the worst thing is that there is a strong group who is able to push their ideas forward although the ideas do not stand reasonable criticism. Most likely this has happened in Ireland although I do not know the background. In Finland a type of Project Maths is used in teaching mathematics but luckily its scope is limited compared to some other countries.*

Unfortunately the comments of Professor Martio seem all too appropriate in the Irish context: a small group has taken control and pushed its ideas. The result, Project Maths, espouses a new philosophy of teaching mathematics at Leaving Certificate. In itself, this would result in significant upheaval as most secondary level teachers do not have the training to teach mathematics in this way. However, in all their wisdom the Project Maths committee has also decided to significantly alter the syllabus. In short, much of calculus (integral and differential) and all linear algebra (vectors and matrices) has been completely removed from the syllabus in order to introduce Euclidean geometry and to beef up probability and statistics. Perversely, this is against the wishes of most secondary school teachers who have years of teaching the omitted topics behind them but will need training to cope with the new topics (geometry in particular). Moreover, despite claiming to have examined the Finnish model, Project Maths is ignoring the experience in Finland. Professor Martio, when speaking of the Finnish experience, also notes that:

> *The curriculum contained concepts the students had no feeling and no background to understand. As mathematicians know statistics is no easy subject and it is my definite opinion that, except discrete probability and some basic continuous distributions, it has no place at the school curriculum. The applied statistics has very much been dropped out in our present curriculum. The Finnish*

> *curriculum still contains some basic faults in teaching probability. Integration, for example, comes too late and without it continuous distributions cannot be understood.*

He also makes this valid point about problem solving emphasis on Project Maths types curricula:

> *Where there is no background, then the problems tend to be so trivial that there is no connection to problems the students meet later. There should be a balance between theory and what is called "problem solving". During the last 40 years the emphasis has been on the side of "problem solving". There is a common belief that this is more useful but this has not been criticised enough*

Indeed upon closer examination of the Finnish advanced syllabus it is clear that calculus and vectors are given very strong emphasis. In addition, there are a number of specialised courses on top of this syllabus, one of which is for example *Advanced differential and integral calculus (MAA13)*, where it is explicitly stated that 'The objectives of the course are, among others, for students to consolidate their knowledge of the theoretical foundations of differential and integral calculus'.

In view of this as well as the comments made by Professor Martio on the inclusion of applied statistics into the mathematics curriculum, it is misleading to suggest that 'the best of this syllabus was taken and adapted to suit the needs of the Irish education systems'.

Is this sensible for Ireland?

Here in Ireland, we are preparing to make the same mistakes already made in Finland. Indeed, at a time when many of these mistakes have since been corrected in Finland, we are heading in the opposite direction and in a more radical way. (Note that, in the above quote, Professor Martio states that 'integration' comes too late: in Project Maths there is no integration at all!) It gets worse. The big users of mathematics in the universities are in engineering, the sciences and business. Calculus, linear algebra, vectors, are their bread and butter. Unfortunately Project Maths has dispensed with much of the mathematics that is regarded as being of key technological and scientific worth (calculus, linear algebra, vectors). Especially noteworthy is the serious reduction in calculus. Discovered by Newton and Leibniz, it is one of the pillars of mathematics, and an absolute requirement for any engineer, scientist or economics student at third level. Yes, these subjects are not easy, but almost every country exposes their students to the intellectual training and rigour of calculus at second level. Instead Irish Leaving Certificate students will study the geometry of Euclid and

some applied statistics. To be sure, most of the geometry examined is of a very elementary nature, and should certainly be taught, but this is not sufficiently challenging for higher-level Leaving Certificate students to be internationally competitive. We stress that the Finnish syllabus, in its various incarnations, has never excluded linear algebra, vectors or calculus.

Korean Experience

In view of the fact that the NCCA also refers to the South Korean syllabus, it may be worth pointing out that their curriculum is in fact vastly more demanding than Project Maths. South Korea is known to have one of the best mathematical education systems in the world. The South Korean second level has a middle school and a high school. At middle school the mathematics curriculum begins with a very strong approach to the teaching of fundamental and advanced algebraic mathematical skills including equation solving. Once this has been completed the focus turns to calculus — mainly differentiation, and this is introduced and developed in a very careful and highly technical manner beginning with concepts such as limits, continuity, and so on. In addition there is a strong emphasis on convergence of series and sequences. After finishing middle school South Korean students move to high school and can take a number of different mathematics courses. The highest level of these is called Mathematics 11, and in this course there is very strong emphasis on integration ending with methods of integration including substitution and integration by parts. We have to conclude, therefore, that Project Maths has not taken the best from the South Korean second level mathematics curriculum as it has significantly reduced the amount of calculus whereas calculus is at the very heart of the South Korean mathematics curriculum. Indeed, there is no comparison between the level of attainment of South Korean students and the Project Maths curriculum.

Mathematics in Canada and Singapore

The mathematics education systems in Canada and Singapore are strongly differentiated. This means that different types of mathematics courses are provided to allow students to achieve at different levels and in different areas of mathematics. A key feature of these curricula is that they require students for example who wish to pursue studies in engineering and science to take particular courses designed to support their mathematical education needs at third level. To take engineering, science and mathematics courses at university in Canada students are

required to take courses in advanced functions and calculus and vectors. Calculus and vectors is regarded as the most demanding of this material.

In Singapore students who wish to pursue engineering and science must take a course called H2. Careful examination of this syllabus again shows a strong technical and in-depth focus on calculus, series and sequences, and linear algebra and vectors. In addition, there is a more demanding syllabus known as H3 which focuses on calculus, combinatorics and mathematical modelling. Indeed in a private communication with one of Singapore's top professors of mathematical education (Professor Peng Yee Lee) on the rationale behind the design of this higher syllabus, he says it is to prepare for university: *'Whether you like it or not, calculus is the gateway to advanced mathematics'.* In this communication he also refers to the aims of the mathematics education system at second level in Singapore:

> *We do not benchmark against TIMSS. In other words, we do not train our students to do well in TIMSS. We do encourage our teachers to set more PISA-like questions for their students*

and

> *We shall have a new syllabus for H3 in 2013. For this year, 2012, we are still using the old syllabus. In fact, before that there were four topics: the current three plus plane geometry. It did not work out. So we dropped geometry. Very often, it is easy to design an intended syllabus, and not so easy to have it implemented.*

It is interesting to note that geometry was dropped and replaced by calculus and mathematical modelling. In another communication, Professor Peng Yee Lee makes the following observation on the approach to teaching mathematics in Singapore: 'Next to modern insights we also hold on to a rigorous approach of *practising*. Problem solving is OK, but you have to be fluent in *basic facts*.'

The Dutch experience

The NCCA also claims to have looked at the situation at secondary level in the Netherlands. Their education system is also highly differentiated and, while there is a roughly equivalent syllabus to Project Maths in existence, there is also a technological or engineering mathematics syllabus, including calculus, linear algebra, vectors *etc.*, which is compulsory for students wishing to study the sciences and engineering.

This salient fact as referred to above has apparently been overlooked by the NCCA.

Conclusions of the International Comparison

What emerges, therefore, from an examination of all the world's top mathematical education systems, is that they are distinguished by a thorough treatment of the very topics, namely calculus (differential and integral), vectors and linear algebra which in Project Maths have been removed to a large extent. Removing this material from the Higher-Level Mathematics Leaving Certificate will result in a very significant lowering of standards of mathematical education at the top level with serious consequences for all further scientific and engineering education and with it for the technological competitiveness of the country.

Clearly, Project Maths ignores key features of differentiated mathematical education systems. Because in the Irish education system there are no additional courses to support technical and scientific education, even our best students will be denied the necessary background for further study in these subjects. Indeed if anything it could be concluded that Project Maths took the lower courses on these curriculums to include it its syllabus.

What appears to have happened in Ireland is that a small group of mathematicians with a very particular approach to teaching mathematics has to taken control (as also in the Finnish experience alluded to by Professor Martio) without proper evaluation, consultation or consideration of the experience in other countries. Despite claiming to wish to "increase applications", the syllabus pointedly omits much of the mathematics at the heart of applications in science and technology. There is no other secondary syllabus in an advanced country which de-emphasises calculus, vectors and linear algebra to the extent that Project Maths does. The introduction of Project Maths, while being detrimental to the scientific education of our own third-level students, will also place them at a disadvantage when travelling abroad. A quick glance at a representative list of questions for entry at Cambridge (see `www.thestudentroom.co.uk`), for example, shows that Project Maths will leave them wholly unprepared. There is little doubt that the watering down of the Leaving Certificate syllabus will have a knock-on effect at third level. A reduction in standards of the same magnitude is *not* occurring in Britain or among any of our European partners. The reputation of our universities will be severely damaged.

Is this really what we want for Ireland? On the face of it, this approach lacks logic and is certainly not consistent with a knowledge-driven or "smart" economy. At best, the new approach is misguided; at worst it is

a small group looking after its own interests and neglecting the needs of the country. The alternative didactic view is that the particular content of the Leaving Certificate syllabus is important in its own right and the earlier that students are exposed to key mathematical concepts (such as calculus and linear algebra) the better.

It is a fact that mathematicians nowadays tend to fall into one of two groups, broadly speaking. Pure mathematicians solve problems arising in mathematics itself; applied mathematicians solve problems arising outside of mathematics (in the sciences, engineering, industry, finance, medicine). Despite the claim that Project Maths makes to "relate mathematics to everyday experience", the syllabus has apparently been largely developed by a small group of mathematicians who have little experience with applications in science or technology: there has been virtually no consultation with the applied mathematics community (or indeed the wider mathematical community).

The remedy

It is the thesis of this group that Project Maths is flawed in the extreme, both as regards the syllabus and as regards the underlying philosophy, which seems to assume that students will be able to analyse real-world problems without the necessary mathematical tools. This will have serious ramifications for students at second and third level as well as the economic competitiveness of the country. The short term solution is to devise a new Higher-Level syllabus with a more balanced and intellectually demanding content including calculus and linear algebra (vectors and matrices, *e.g.* the equation of a plane in three dimensions in vector form). This group would be willing to work out a proposal for such a syllabus if required, which could be offered to schools as an alternative to Project Maths. A long term solution must include dealing with the issue of teacher training for second level as well as the introduction of suitable courses at all levels to support educational needs in mathematics at third level.

Consequences for Ireland if Project Mathematics if left unchanged

Mathematics is an underpinning technology at the heart of any smart economy. Project Mathematics comprises both a new approach to teaching and a new mathematics syllabus at secondary level. It is seriously deficient. Notably,

(1) Project Mathematics is flawed in the extreme in its syllabus and methodology.

(2) Project mathematics will result in very significant lowering of standard in the second level mathematics education system. This will have detrimental long term impacts on scientific and engineering educations systems at all levels.

(3) Project mathematics will leave students wholly unprepared for the study of engineering, science and mathematics at third level.

(4) Project mathematics will severely damage the reputation of our universities and institutes of technology.

(5) Project mathematics will seriously damage international reputation of our whole scientific educational system.

(6) Project mathematics will seriously damage Ireland's international competitiveness.

(7) Project mathematics will not provide sufficiently technically capable graduates, with the requisite skills and competencies to support further development of the 'smart' or 'knowledge' economy.

Conclusions and Recommendations

(1) The current Project Mathematics Higher-Level Leaving Certificate course is completely insufficient and unsuitable by international standards to support top level engineering, scientific and mathematical education. This is because topics (calculus, series and sequences, linear algebra and vectors) which lie at the heart of all the worlds top curricula on similar courses, and which were at the heart of the previous Leaving Certificate syllabus) have been removed in large measure by Project Mathematics curriculum design team. This was a very serious mistake by Project Mathematics design team. It is therefore imperative that new mathematics courses along the lines suggested at Higher Level and (even also at Ordinary Level) be introduced as a matter of urgency to support these areas. Otherwise the consequences for our mathematical engineering and scientific education systems at all levels will be very serious.

(2) Project maths will result in a very significant lowering of standards in the Irish mathematical education system at second level. This will have serious consequences not only for the mathematical education itself but also for the scientific and engineering education systems at all levels. This will also result in very serious consequences for development of technical competencies and skills required to support 'smart' and 'knowledge' economy.

This should be a matter of very serious concern for all involved in mathematical education and policy makers.

(3) In views of statements made by Project Maths which purport to show that Project Maths design team took the best of a number of world's top mathematics curriculums and tailored them to suit the Irish mathematical education system at second level, the evidence presented proves that this was simply not the case. Indeed claims made by NCCA are entirely misleading. In view of these there appears to be very serious questions to be asked of the NCCA curriculum design processes. Unquestionably vast amounts of money have been spent on the introduction of this new syllabus into the Irish mathematical education system. The only conclusion that can be made is that these processes resulted from completely insufficient, and inadequate appropriate curriculum design expertise, at top level in NCCA in mathematics, and from higher education representatives on the Project Maths mathematics design team. This is a very serious matter for the NCCA and the minister for education.